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Packet radio networks that employ several parallel multiple-access channels are considered. An architecture for such a network dictates the selection of channels for packet transmission. We propose and analyze two multichannel architectures. In the first, each node employs a single radio and is assigned a channel on which it listens when it does not transmit. To transmit a packet, the node tunes its radio to the channel of the intended receiver for that transmission only. The second architecture requires each radio to use a single channel for both transmission and reception, but provides some of the nodes with more than one radio each, allowing them to serve as bridges between channels. Within these architectures one can further select the amount of routing information held by each node and the channel-access protocol, both of which greatly affect the network performance. To ascertain the effect of the various parameters, we calculate the

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I. INTRODUCTION

Most of the research and development of packet radio networks (PRNET) has been directed so far toward producing networks in which all nodes share a single channel [1, 2, 3]. In such PRNETs, when two or more packets arrive at a receiver with comparable power during overlapping intervals, none of them is received correctly. Working with a single channel is a natural mode of operation for PRNETs since they use radio transmitter-receivers (transceivers) that can be tuned to one channel at a time. However, when the network has to carry a high level of traffic, the single channel quickly becomes the bottleneck for network performance.

An alternative mode of operation for a PRNET makes several channels available to the nodes. In this multichannel mode, packets that are transmitted simultaneously on different channels cause little or no interference to one another. When the channel signalings are orthogonal, as is the case in frequency division multiple access (FDMA), a packet transmitted on a given channel is received successfully only if the receiver is tuned to that channel and if there is no interference from other packets on the channel.

An important advantage of the multichannel mode is that the PRNET can increase or decrease its capacity by adding or deleting channels. This flexibility allows one to construct PRNETs that can be adapted to higher levels of traffic without modifying the radio hardware. Thus, a set of multichannel radio networks can share more efficiently the available RF spectrum than single-channel networks since, in the latter case, each network uses the same bandwidth regardless of its traffic volume.

Some studies of multichannel multiple-access protocols can be found in the literature. Yung [4] analyzed a single-hop, slotted ALOHA protocol in which each of the nodes has several transmitters. Each node can transmit simultaneously on as many channels as it has transmitters, with all the transmissions directed towards a central station. Multichannel, carrier-sense multiple access with collision detection (CSMA/CD) local-area networks (LAN) have also been analyzed [5, 6, 7]. These papers have shown that the throughput of this type of network increases when the bandwidth is split into several narrower-band subchannels. Since, for a fixed packet length, the ratio of propagation delay to packet transmission time is smaller on each of the subchannels than it is on the wide channel, each subchannel has a higher efficiency than that achieved by using the whole bandwidth as a single channel. A similar improvement can be anticipated in a multihop PRNET. However, since the effectiveness of multihop CSMA is rather limited because of interference from hidden nodes, it is not clear that splitting the channel will improve performance.

Multihop, multichannel PRNETs have been studied under the code-division multiple-access (CDMA) protocol, in which the network operates with multiple, parallel, spread-spectrum codes [8, 9]. Some of the material presented here would probably apply to CDMA too; however, consideration of that protocol's specific constraints, such as cross-channel interference, is outside the scope of this paper.

In this paper we consider multihop PRNETs that use several orthogonal channels. We investigate two architectures for such networks. In the first architecture each node employs a single transceiver and is assigned a channel to which it is tuned when it is not transmitting. To transmit a packet, the node tunes its radio to the channel of the intended receiver, a technique called *receiver-directed* transmission [1]. The second architecture requires each radio to remain always on its designated channel, but provides some of the nodes with more than one radio each so they can serve as bridges between channels. Both architectures can use the same channel-access protocols as single-channel networks. In addition, these networks can operate under vari-

ous levels of routing information depending on the channel overhead that can be tolerated. These architectures are described in Section II.

We discuss in Sections III and IV the performance of these schemes. We first analyze the receiver-directed architecture, where the performance is measured by both the throughput per node and the expected progress per hop for a packet [10]. This architecture is analyzed under slotted ALOHA and CSMA protocols both with and without capture. The effects of routing information available at the nodes is also ascertained. The second architecture is then analyzed under the slotted ALOHA protocol. In section V we discuss the numerical results and compare the performance of the various schemes analyzed in this paper.

II. ARCHITECTURES AND ROUTING

As indicated above, multichannel PRNETs can be constructed in a variety of ways and, when the choice of channel-access protocols is taken into account, it is clear the number of possible network versions can be very large. To focus the discussion we have selected a set of typical architectures that are analyzed in this paper. These architectures are categorized by the number of transceivers per node, that is, the number of channels on which a node can transmit/receive simultaneously. The first subsection is devoted to schemes that require only one radio per node, the same hardware requirement as in a single-channel PRNET. We then describe an architecture where some of the nodes have more than one radio. This requires more hardware than the previous scheme but provides for better partitioning of the nodes and allows packets to be transferred between channels only at a well defined set of nodes. These features are required for some military applications.

A. An Architecture with a Single Transceiver per Node

We denote the channel to which a node is tuned when not transmitting as its *quiescent* channel. When a node wants to send a packet to a neighbor, it tunes its transceiver to the quiescent channel of the intended receiving node and transmits on that channel. Following a transmission, the transceiver is retuned to its own quiescent channel.

The receiver-directed scheme allows the PRNET to operate on multiple channels without any increase and/or modification to the hardware, compared to a single-channel network. However, the broadcast capability of the single-channel network, which allows a node to reach all its neighbors with a single transmission, no longer exists in the receiver-directed scheme. Local broadcasting can be achieved, for example, by repeating the packet transmission on all the transmitter neighbors' quiescent channels. The lack of local broadcast will have a major effect on the distribution of routing information: currently, PRNET disseminates routing information by having each node periodically broadcast its routing tables [2]. In a multichannel PRNET, the same amount of routing information can be obtained by broadcasting the routing information on the quiescent channel of each of the node's neighbors. In this case the size and contents of the routing tables are similar for single and multichannel PRNETs. This type of information allows a node to find the shortest path to any destination: thus it transmits its packet on the quiescent channel of the next node of that path. We denote this scheme as the *full-information* routing scheme.

The necessity for multiple transmissions of each routing update under the full-information routing scheme, implies a higher overhead for the network than in a single-channel network. This higher cost in channel resources increases with the number of channels employed by the network. It has a specially strong effect in dense networks where each node has many neighbors, and, since each of the nodes broadcasts its routing information, the proportion of the channel devoted to routing traffic tends to be high.

A reduction in this overhead can be achieved if each node broadcasts its routing table only on its own quiescent channel. This limits a node's routing information only to nodes on the same quiescent channel and its immediate neighbors on other channels. Thus, the routing tables are smaller, and, since they are transmitted fewer times, the overhead traffic is less than in the full-information routing scheme mentioned above. Under this limited routing scheme, the PRNET will be practically partitioned into clusters of nodes sharing the same quiescent channel.

By occasionally listening to other channels, a node can discover all its neighbors whose quiescent channel is different from its own. Thus, when a node wishes to pass information to a node on another channel, it first transmits the packet to a neighbor that has the same quiescent channel as the destination (by tuning to that channel for that transmission). From that neighbor the packet will be routed using the routing information available to the nodes of the destination's quiescent channel. Although a node does not know the whereabouts of nodes that are neither neighbors nor belong to the same channel, we assume that it does know the quiescent channel of the destination. Since this information does not change frequently, its acquisition and maintenance imposes little overhead upon the network.

In a dense network a node may have more that one neighbor of the destination's channel. In this case the node faces the issue of which neighbor to choose as a transfer point. Two choices, namely select a transfer point at random, or select the nearest transfer node, will be analyzed. We denote these schemes as *partial-information* routing schemes.

Notice that this local routing information is sufficient to route from any node in the network to any other node provided that two conditions are satisfied: (1) each node has at least one neighbor on every channel, and (2) the set of nodes in each channel constitute a connected graph, i.e., between any two nodes of the same channel there is a path that traverses only nodes of that channel. These two conditions are likely to be satisfied in dense networks where each node has a large number of neighbors. Since the multichannel scheme is aimed mainly at dense networks, these conditions are not overly restrictive. Furthermore, the network can still operate even if the

above conditions are somewhat relaxed. For example, if each cluster has at least one neighbor from every other channel, the list of neighbors can be carried in the routing update message so that a packet to a node on a different channel is first carried to a neighbor of the cluster and from there as before to the destination. If the second condition is not satisfied, i.e., a packet cannot reach every destination by, at most, one channel transfer, then techniques similar to those in hierarchical networks [11] can be used. However, for the sake of analytical simplicity, we will consider only networks that satisfy both conditions.

B. Network Partition with Bridges

The receiver-directed schemes described above are very flexible in the sense that every node can transmit to any of its neighbors. Thus, traffic can cross to a different channel at any point in the network. It is sometimes desired, mainly in military networks, to limit the nodes' flexibility to access the channels in such a way that each radio has to stay only on its designated channel, both for transmission and for reception.

To allow for packet transfer between channels, some of the nodes are equipped with multiple radios each, so that these nodes can transmit and receive on as many channels simultaneously. These special nodes serve as bridges between the channels; that is, when a node wishes to send a packet to a destination node on another channel, it has to transmit it via a bridge.

This scheme can operate with limited routing information, as described above, where each node keeps the routing information about the set of nodes that belong to the same channels as its own. Each node also keeps information regarding the bridges and the channels they lead to. Notice that a bridge keeps routing information about all the channels it has transceivers on. The advantages of this scheme are the resulting network partition and the ability of the bridges to operate on several channels simultaneously. These advantages, however, come at the cost of extra hardware at some of the nodes.

The above schemes, both the receiver-directed and the one with bridges, can be extended to include multiple radios at all nodes, which implies that each node can operate on several channels at the same time. Adding radios to nodes increases the number of possible variations of the architecture and the number of channel-access protocols that can be used. A full discussion of these possibilities will form the subject of a forthcoming paper.

III. PERFORMANCE EVALUATION

In the previous section we saw that both multichannel architectures, the receiver-directed and the network partition with bridges, allow for packet transfer between any two nodes in the PRNET. Notice that both architectures are flexible in the number of channels they can use. A question thus arises as to how much improvement in network performance is gained by adding channels. To be able to answer this question quantitatively, we develop analytical models for the performance of multihop multichannel PRNETs that operate under the schemes described above and various channel-access protocols and environments. Our model is an extension of the one first suggested by Silvester [12] and subsequently used by several other authors [10, 13, 14]. The model's basic assumption is that the PRNET nodes are scattered, according to a Poisson process on the plane, over a large area. The model allows the analysis of a typical node in the interior of the network so that edge effects can be ignored. It is also assumed that the time is slotted, and, in each slot, the node population is redistributed independently of previous slots.

A. Receiver-Directed Transmissions with Partial Routing Information

Let us denote by C the number of channels used by the network and by λ the parameter of the Poisson process controlling the nodes' distribution in each channel. Transmission range is assumed to be constant, denoted by R . A packet has to traverse, on the average, L hops from source to destination if it is transmitted on the optimal route. This means that, under the partial-information routing scheme, packets intended for radios of the same channel make L hops, on the average. Packets ultimately destined for radios of some other channel make $L+1$ hops, on the average, since, on their first hop, packets are routed in a random direction. Because the source does not know the destination's location, it cannot select the next repeater on the route. This first-hop transmission thus contributes no progress. The next node, however, which is on the destination's channel, knows the location of the destination and therefore forwards the packet on the best path within their common channel.

The probability that a newly generated packet is destined for a radio of any particular channel is assumed to be uniform: $\frac{1}{C}$. We observe that a packet directed to a node on the same quiescent channel as the transmitting radio will have a probability of successful reception different from that of a packet directed to a radio of a different quiescent channel because of the different routing used and, consequently, the different expected interference. The first type is denoted here as ***in-channel*** traffic, the other type as ***cross-channel***. Because our model considers the network at a random slot, in order to calculate the probability of success, it is necessary to know the conditional probability that a node transmits on a particular channel, given the fact that it transmits in that slot. We begin by calculating this probability.

Let s_i and s_c be the conditional probabilities that in-channel and cross-channel transmissions, respectively, are successful, given that such transmissions take place. From these we can calculate the conditional probabilities that a node transmits to a particular channel, given that that node transmits. Knowing the probabilities of transmission in a particular channel, we can calculate the probabilities of success in that channel. Since, in general, the probability of success for the in-channel transmissions depends on cross-channel transmissions from other channels, we

have a fixed point relationship, $s_i = f_i(s_i, s_c)$ and $s_c = f_c(s_i, sc)$, that these probabilities must satisfy.

The conditional probabilities of transmission to a particular channel, given that a transmission takes place, can be found independently of the protocol being used. We use arguments that are based on the long-run proportions of the number of occurrences of different types of packet transmission. On a hop where the success probability is s, a packet will be transmitted on the average $1/s$ times. Since newly generated packets are uniformly addressed to radios of all channels, a proportion $\frac{1}{C}$ of them will be transmitted in-channel, while the other $\frac{C-1}{C}$ will be transmitted cross-channel. The packets that were transmitted in-channel on their first hop will be transmitted an additional $L-1$ times in-channel; those that made their first hop cross-channel will make L more hops in-channel.

Since, on its first hop, a packet goes to any of the C channels with equal probability, the average number of times a randomly selected packet is transmitted on its first hop is $\frac{1}{C}\left(\frac{1}{s_i}+\frac{C-1}{s_c}\right)$. Subsequent hops are made in the channel assigned to the final destination, so the average number of transmissions per hop is $\frac{1}{s_i}$. The probability that a packet observed at a random slot is making its first hop is then given by

$$P(\text{first hop}) = \frac{\dfrac{1}{s_i}+\dfrac{C-1}{s_c}}{\dfrac{1}{s_i}+\dfrac{C-1}{s_c}+\dfrac{L-1}{s_i}+\dfrac{(C-1)L}{s_i}} = \frac{\dfrac{1}{s_i}+\dfrac{C-1}{s_c}}{\dfrac{CL}{s_i}+\dfrac{C-1}{s_c}} \qquad (1)$$

Now the probability that a packet on its first hop is being transmitted cross-channel and to a specific channel is

P(cross-channel to a specific channel | first hop)

$$= P\,(\text{to a specific channel} \mid \text{cross-channel and first hop})$$

$$\cdot P\,(\text{cross-channel} \mid \text{first hop})$$

$$= \frac{1}{C-1} \cdot \frac{\dfrac{C-1}{s_c}}{\dfrac{1}{s_i} + \dfrac{C-1}{s_c}} \qquad (2)$$

and hence the conditional probability that a packet is being transmitted cross-channel to a specific channel is

$$h_c = \frac{\dfrac{1}{s_c}}{\dfrac{CL}{s_i} + \dfrac{C-1}{s_c}} \qquad (3)$$

The conditional probability that a packet is being transmitted in-channel is then

$$h_i = P\,(\text{in-channel,first-hop}) + P\,(\text{in-channel,other-hop})$$

$$= P\,(\text{in-channel,first-hop}) + (\,1 - P\,(\text{first-hop}))$$

$$h_i = \frac{\dfrac{1}{s_i}}{\dfrac{CL}{s_i} + \dfrac{C-1}{s_c}} + \frac{\dfrac{CL-1}{s_i}}{\dfrac{CL}{s_i} + \dfrac{C-1}{s_c}}$$

$$= \frac{\dfrac{CL}{s_i}}{\dfrac{CL}{s_i} + \dfrac{C-1}{s_c}} \qquad (4)$$

Now that we know the conditional probabilities of in-channel and cross-channel transmissions, the probability of success for in-channel transmissions, s_i, and for cross-channel transmissions, s_c, can be computed. The calculations are not only different for slotted ALOHA and CSMA, but also require modification when FM capture conditions are allowed for.

1. Slotted ALOHA

In the slotted ALOHA protocol, each radio will transmit in a particular slot with probability p, independently of all other radios. A transmitted packet will be received correctly if no other radio within range of the receiver transmits on the same channel in the same slot. Thus for a transmission in channel i, say, the probabilities of interference by another radio, if it is within range, is

$$P(\text{channel } i \text{ interferes}) = p \ h_i = q_i \tag{5}$$

$$P(\text{channel } k \neq i \text{ interferes}) = p \ h_c = q_c \tag{6}$$

Using Eqs. (5) and (6), the probability of success for an in-channel transmission can be calculated. The transmission will succeed if (a) the receiver does not transmit on any channel (probability $1-p$) and (b) none of the other radios within range of the receiver transmits on its channel. Consider the "most forward within R" (MFR) routing rule [10], in which the transmitter selects the next node in the route such that the maximum progress towards the final destination would be achieved if the transmission succeeded. Assuming that this selected receiver is at distance r from the transmitter and the direction to it is at angle θ relative to the direction to the destination (see Figure 1), the conditional probability of success is as follows:

$$s_i(r,\theta) = (1-p) \sum_{j=0}^{\infty} (1-q_i)^j \ P(j \text{ radios in } A(r,\theta)) \cdot \left[\sum_{k=0}^{\infty} (1-q_c)^k \ P(k \text{ radios in } B(r,\theta)) \right]^{C-1} \tag{7}$$

$$s_i(r,\theta) = (1-p)\, e^{-\lambda q_i A(r,\theta)}\, e^{-\lambda q_c (C-1) B(r,\theta)} \quad , \tag{8}$$

where $A(r,\theta)$ is the area within which a radio whose quiescent channel is the same as the receiver's may interfere with transmissions to the receiver at (r,θ), if it transmits. $B(r,\theta)$ is the equivalent area for radios of a different quiescent channel. The area A is a circle of radius R around the receiver, with the area labeled X in Figure 1 excluded as it is known, because of the "most forward" routing, that there are no radios of the same channel in X. The area involved is given by Hou and Li [14], or it can be deduced from our results for the cases of capture that are given in the appendix. B is merely a circle with radius R since the transmitter knows nothing about the distribution of the radios in the other channels.

Similarly, if we transmit to a random cross-channel neighbor, the area of interference affecting cross-channel traffic is a circle for radios of all channels.

$$s_c(r,\theta) = (1-p) \sum_{j=0}^{\infty} (1-q_i)^j\; P(j \text{ radios in } B)$$

$$\cdot \left[\sum_{k=0}^{\infty} (1-q_c)^k\; P(k \text{ radios in } B) \right]^{C-1} \tag{9}$$

$$s_c(r,\theta) = (1-p)\, e^{-\lambda p \pi R^2} \tag{10}$$

The average successful traffic is found by integrating $s_i(r,\theta)$ and $s_c(r,\theta)$ over the possible values of r and θ, and including the probability of making the appropriate type of transmission. We assume that a transmission is only made if there is a receiver of the appropriate channel in range, and in a forward direction if the transmission is to be in-channel. If we choose the node that makes the most forward progress for in-channel transmissions, then the density of (r,θ) for the receiver is (see Hou and Li [14])

$$f(r,\theta) = \frac{\lambda\, r\; e^{-\lambda R^2(\cos^{-1}t \,-\, t\sqrt{1-t^2})}}{1 - e^{-\lambda R^2 \pi/2}} \quad \text{, where } t = \frac{r\cos\theta}{R}. \tag{11}$$

Therefore,

$$s_i = \int_0^R \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} s_i(r,\theta)\, f(r,\theta)\, d\theta\, dr \tag{12}$$

For cross-channel transmissions, the density is uniform, given that there is a receiver within range, so

$$s_c = \frac{(1-p)\, e^{-\lambda p \pi R^2}}{1 - e^{-\lambda \pi R^2}} \tag{13}$$

We now have a set of nonlinear equations in s_i. (In this case s_c is constant.) These can be solved by using a simple iteration, $s_{i\ new} = f\left(s_{i\ old}\right)$. The initial value of s_i is taken from the [exact] solution to the single-channel case. Only a small number of iterations were required for this procedure to converge.

The in-channel throughput S_i, which is defined as the average number of successful in-channel transmissions per node per slot, is given by

$$S_i = q_i \left(1 - e^{-\lambda R^2 \pi/2}\right) s_i \quad , \tag{14}$$

and the cross-channel throughput, defined similarly, is

$$S_c = q_c\ (C-1) \left(1 - e^{-\lambda \pi R^2}\right) s_c \quad . \tag{15}$$

The expected progress that a packet transmitted in-channel to a receiver at (r,θ) makes toward its final destination is

$$z(r,\theta) = r\ \cos\theta\ s_i(r,\theta) \quad . \tag{16}$$

Notice that the cross-channel traffic, on the average, makes no progress. The expected forward progress per node per slot is given by:

$$Z = q_i \left(1 - e^{-\lambda R^2 \pi/2}\right) \int_0^R \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r\ \cos\theta\ s_i(r,\theta)\, f(r,\theta)\, d\theta\, dr \tag{17}$$

When specialized to a single-channel network, these equations agree exactly with those of Hou and Li [14].

a. Modification for Nearest-Neighbor, Cross-Channel Transmissions

A transmitter is not completely ignorant of the distribution of radios in other quiescent channels; it knows those that are its neighbors. An improvement in throughput may be possible if a node selects the nearest neighbor, rather than a random one, of the appropriate channel, for cross-channel transmission. The intuition behind this selection is that the area about which we know nothing is that outside the radius of transmission. By reducing this, we reduce the expected interference. We also know that a circle centered on the transmitter has no radios of the same quiescent channel as the receiver. Referring to Figure 2, assuming that Q is P's nearest neighbor, we see that there are no radios of the same quiescent channel as Q in the circle radius r, centered at P. The area containing radios that can potentially interfere at Q is the circle radius R centered at Q with the smaller circle removed. The area that is known to have no radios is labeled X in Figure 2. Thus the success probability for a cross-channel transmission is given by:

$$s_c(r,\theta) = (1-p) \sum_{j=0}^{\infty} (1-q_i)^j \, P(j \text{ radios in } D) \left[\sum_{k=0}^{\infty} (1-q_c)^k \, P(k \text{ radios in } B) \right]^{C-1} \quad (18)$$

The area D depends on the distance r from transmitter to receiver. The area involved is calculated in the appendix. To find the unconditional s_c we need to integrate this function taking into account the density of the nearest neighbor;

$$g(r,\theta) = \frac{\lambda r \, e^{-\lambda \pi r^2}}{1 - e^{-\lambda R^2 \pi}}.$$

$$s_c = \int_0^R \int_{-\pi}^{\pi} s_c(r,\theta) \, g(r,\theta) \, d\theta \, dr$$

The iteration used to solve the fixed point relationship for s_i and s_c now involves both, but still converges very fast. The initial value of s_c is taken from the [exact] solution to a single-channel model using the transmit-to-nearest routing protocol.

2. CSMA with MFR Routing

In single-hop fully connected PRNETs, carrier-sense protocols improve throughput dramatically. In this protocol, we assume that the time is partitioned to minislots, each of which is about the length of the maximum propagation delay between two neighboring nodes, a delay that is usually smaller than the packet transmission time. A radio wishing to transmit senses the channel and, if a transmission is in progress, the radio waits until the transmission is over before attempting to reschedule its packet. At the next sense instant, if the channel is idle, the radio transmits with probability p or waits for another slot and senses again with probability $1-p$. Notice that due to possible hidden nodes, sensing the channel idle by the transmitter does not guarantee that the channel is idle at the receiver.

Again we assume that the routing is to the "most forward within R." Takagi [10] has analyzed single-channel CSMA in a multihop environment. Following Takagi's approach, we assume that a packet transmission time is our unit of time and that the length of the slot is a. We define $\tau=\frac{1}{a}$, and assume τ is an integer. Although, because of the sensing, packet transmissions do not start independently in every slot, we assume that a packet transmission starts with probability p' in any otherwise idle slot. This assumption that transmission starts form an independent Bernoulli process has been shown by simulation to be a reasonable approximation [15]. In the following we assume partial-information routing with random neighbor selection for cross-channel transmission. However, the extension to nearest neighbor selection or full-information routing is straightforward.

Consider the situation depicted in Figure 3 where a transmission is attempted by node P to Q; the radios that can interfere with this transmission are in a circle, radius R, centered on Q. Those that are also within radius R of P will be able to sense P's transmission in progress, so they will interfere only if they start a transmission in the same slot. Those that are farther from P than R cannot hear P's transmission, so any transmission they may begin in the next τ slots will interfere; conversely, P cannot hear any of them, so that, if any of them begin transmitting in any of the preceding τ slots, P's transmission will be interfered with; this is the area labeled $Y(r,\theta)$ in Figure 3. Furthermore, Q must not start a transmission in the same slot as P. Thus we have

$$P\,[success\ \mathrm{P}\rightarrow\mathrm{Q}] = Prob\,[\mathrm{Q}\ does\ \ not\ \ start\ \ a\ \ transmission\ \ in\ \ slot\,]$$

$$\cdot Prob\,[No\ \ transmission\ ALOHA\ \ mfr\ \ area\ \ \mathrm{for}\ 1\ slot\,]$$

$$\cdot Prob\,[No\ \ transmission\ Y\ \ area\ \ \mathrm{for}\ 2\tau\ slots\,]$$

$$s(r,\theta) = (1-p')\, e^{-\lambda p' A(r,\theta)}\, e^{-\lambda p' 2\tau Y(r,\theta)} \quad , \tag{19}$$

where p' satisfies the following relationship [10],

$$p' = \frac{a\ p\ e^{-p' N}}{1+a-e^{-p' N}} \ .$$

When we consider a multichannel case, the radio transmits to the receiver "most forward within R" if the packet is destined to a radio of the same channel, and to a random receiver of the correct channel if the packet is for a different channel. The same analysis we used for the multichannel slotted ALOHA can be used to give an iterative procedure for finding s_i and s_c.

The procedure for estimating the conditional probability of transmitting in-channel and cross-channel, given that a transmission occurs, is the same as in the ALOHA case. The analysis of CSMA given above applies to the in-channel traffic directly. Cross-channel transmissions can interfere if they start in the same slot, for any radio within a circle of radius R centered at the receiver. They will also interfere if any radio in the Y area starts transmission for 2τ slots.

$$s_i(r,\theta) = (1-p') \, e^{-\lambda q_i (A(r,\theta) + 2\tau Y(r,\theta))} e^{-\lambda q_c (C-1)(\pi R^2 + 2\tau Y(r,\theta))} \tag{20}$$

where $q_i = p' h_i$, and $q_c = p' h_c$, and the cross-channel success is given by

$$s_c(r,\theta) = (1-p') \, e^{-\lambda p' (\pi R^2 + 2\tau Y(r,\theta))} \quad . \tag{21}$$

As before, we find the unconditional values of s_i and s_c by integration. The throughput is given by the same formula as before.

3. FM Capture

FM radios have the ability to receive a transmission successfully in the presence of weaker transmissions on the same frequency. This phenomenon is known as *capture*, and can contribute significantly to the throughput of a packet radio network.

We model the capture phenomenon by using the notion of a *capture parameter*, α, which was introduced by Roberts [16] and used by many subsequent authors[17, 18, 19, 10]. That is, a receiver, Q, will receive a packet transmitted by P at distance r correctly if and only if no radio within distance αr is transmitting simultaneously. The value of α can be as small as 1, for perfect capture, and as large as ∞, implying no capture at all. We continue to assume that a radio has no effect outside its radius of transmission, R, so that the area containing potentially interfering radios is a circle of radius r', where $r' = \min[\alpha r, R]$.

For slotted ALOHA, the area that may contain radios that generate in-channel interference is a circle, radius r', center Q, excluding the area known to be empty because of the "most forward" routing. In the multichannel case, the interference caused by radios from other channels comes from a circle whose radius is r'.

Similarly, in CSMA the area of radios that cannot sense the transmission in progress but can interfere is a circle of radius r' around the receiver, with the circle of radius R around the transmitter removed.

For details of these areas and the formulas used to calculate them, see the appendix.

B. Receiver-Directed Transmissions with Full Routing Information

In our analysis so far we have considered only receiver-directed schemes with partial-information routing schemes. In this section we evaluate the performance of a multichannel PRNET where each node has full routing information regarding all the nodes in the network. The main advantage of this scheme is that the node can route under MFR from the first hop on, regardless of the channel it is sent on.

If we assume full knowledge of the positions of the radios of all channels, and allow the possibility that the partitioning of radios between channels is not uniform and that transmissions to different channels may be made at different powers, the analysis becomes much more involved.

We present it here for the case of two channels, although the generalization to many channels is conceptually straightforward. Assume that radios in channel i, $i=1,2$, are distributed as a Poisson process in the plane with parameter λ_i and that transmissions to channel i have an effective radius R_i. Without loss of generality we assume that $R_1 \leq R_2$. The routing protocol is assumed to be most forward within R_2. The probability of finding at least one receiver in the forward direction is $P_{forward}$.

$$P_{forward} = 1 - e^{-\frac{\pi}{2}(\lambda_1 R_1^2 + \lambda_2 R_2^2)}$$

The density function of the position of the receiver under this protocol is quite complex. Referring to Figure 4, if the receiver is in the area labeled "a," it must be of type 2, with no type 2 nodes further forward. In this case we know nothing about the distribution of type 1 nodes. In area "b," the radio must be of type 2, but as well as knowing that there are no type 2 radios further forward within R_2, we know that, within R_1 of P, there are no type 1 nodes farther forward than our chosen receiver. In area"c," the receiver can be of either type with probability proportional to the respective densities. Using the same technique as Hou and Li [14], we find that

$$P_{forward} \cdot f(r,\theta) = \begin{cases} (\lambda_1 + \lambda_2) r e^{-\lambda_1 A_s(r,\theta,R_1) - \lambda_2 A_s(r,\theta,R_2)} & r\cos\theta < R_1 \text{ and } r < R_1 \\ \lambda_2 r e^{-\lambda_1 A_s(r,\theta,R_1) - \lambda_2 A_s(r,\theta,R_2)} & r\cos\theta < R_1 \text{ and } r \geq R_1 \\ \lambda_2 r e^{-\lambda_2 A_s(r,\theta,R_2)} & r\cos\theta \geq R_1 \end{cases}$$

where

$$A_s(r,\theta,R) = R^2(\cos^{-1}t - t\sqrt{1-t^2}) \quad t = \frac{r\cos\theta}{R}$$

The probability that a transmission will be to a radio of channel 2, given that a transmission is made, is

$$q_2 = \int_{R_1}^{R_2} \int_{-\pi/2}^{\pi/2} f(r,\theta)\, d\theta\, dr + \frac{\lambda_2}{\lambda_1 + \lambda_2} \int_{0}^{R_1} \int_{-\pi/2}^{\pi/2} f(r,\theta)\, d\theta\, dr$$

and that it will be to a radio of channel 1 is

$$q_1 = \frac{\lambda_1}{\lambda_1 + \lambda_2} \int_{0}^{R_1} \int_{-\pi/2}^{\pi/2} f(r,\theta)\, d\theta\, dr$$

Although the probabilities of transmission to radios of different types are different, so are the success probabilities. Although these two effects work against each other, that is although radios of type i have a higher probability of being transmitted to, these transmissions are less likely to be successful than those to radios of the other type.

The success probability depends on which type of radio the transmission is made to. Any transmission to a radio at a distance greater than R_1 *must* be to a radio of type 2, whereas, if the intended receiver is within R_1 of the transmitter, the radio will be of either type with probability proportional to their respective densities. The area of potential interfering radios is different in the three regions where the density function $f(r,\theta)$ is different. When the forward progress is greater than R_1 (area "a" in Figure 4), we know nothing about the distribution of type 1 radios around the receiver, so the success probability is

$$s(r,\theta) = (1-p)e^{-pq_2\lambda_1\pi R_2^2}\ e^{-pq_2\lambda_2 A(r,\theta,R_2,R_2)}, \quad r\cos\theta \geq R_1$$

$A(r,\theta,\sigma,t)$ is the area containing radios that may interfere with the receiver at (r,θ). t is the radius of the circle around the receiver within which they all must lie. σ is the radius of the circle within which it is known that this radio is the most forward. Note that, because of the two different transmission radii, the receiver can lie outside the circle of radius R_1 and still imply that there is a region inside that circle that has no radios. This is the same area involved in the case of MFR with capture when $t \leq R$, and the same formula applies when $t > R$. When $r \geq R_1$, but the progress is less than R_1 (area "b" in Figure 4), we know that we are transmitting to a radio of type 2, but we also know that there are no type 1 radios farther forward and within R_1 of the transmitter.

$$s(r,\theta) = (1-p)\ e^{-pq_2\lambda_1 A(r,\theta,R_1,R_2)}\ e^{-pq_2\lambda_2 A(r,\theta,R_2,R_2)}, \quad r\cos\theta < R_1 \text{ and } r \geq R_1$$

When the receiver is closer than R_1 (area "c" in Figure 4), the interference depends on whether the receiver is type 1 or type 2.

$$s(r,\theta) = (1-p)\,\frac{\lambda_1 e^{-pq_1(\lambda_1 A(r,\theta,R_1,R_1)+\lambda_2 A(r,\theta,R_2,R_1))} + \lambda_2 e^{-pq_2(\lambda_1 A(r,\theta,R_1,R_2)+\lambda_2 A(r,\theta,R_2,R_2))}}{(\lambda_1+\lambda_2)},$$

$$r\cos\theta < R_1 \text{ and } r < R_1.$$

As before, the average throughput is found by integrating this function, and the average progress can be found by incorporating the term $r\cos\theta$ in the integration.

When this model is specialized to the symmetric case, $\lambda_1 = \lambda_2$ an $R_1 = R_2$, the analysis becomes much simpler and is easily generalized to the C channel case. The density function of the receiver's position is the same as that of a single-channel system using MFR, except that the parameter of the Poisson process is $C\lambda$; the density of the radios that can interfere is still given by λ. The choice of most forward receiver is made from a dense network, but the interference comes from a sparse network.

C. Network Partition with Bridges

In this section we analyze the performance of this scheme with partial-information routing, slotted ALOHA as the channel-access protocol, and each bridge operating on all channels simultaneously. A regular node, of course, operates on one channel only. As before, the extension of the analysis to CSMA, or to other routing schemes, is straightforward. C denotes the number of channels. For analytic simplicity we consider a symmetric system in which the regular nodes of each channel and the bridges are distributed in the plane as a Poisson process with rate λ_r and λ_b, respectively. In each slot a regular node transmits with probability p_r and a bridge with probability p_b on each one of its channels.

A newly generated packet is destined to any other node with equal probability. From this fact, the distribution of bridges and regular nodes, and the assumption that a packet's average route is L hops on its destination's channel (when it is routed according to MFR), we can calculate the conditional probability that, given a regular node transmits, its transmission is destined to a node of the same channel or of a different channel. We shall denote these as in-channel and cross-channel transmissions, respectively. In-channel transmissions are sent by MFR, whereas a cross-channel transmission is sent to a bridge at random. Both types of transmission take place in the channel of the transmitting node; it is the different routing policies that make the success probabilities different. We begin by calculating P_{in}, the conditional probability that a node transmits in-channel given that it transmits.

In each unit area there are λ_r regular nodes on each channel and λ_b bridges. We assume that all nodes in the network generate new packets at the same rate. Our objective is to find the ratio of packets *transmitted* by a regular node that are sent in-channel. Thus, at a regular node, for each $\lambda_r + \lambda_b$ new packets that can be sent without changing channel, there are $(C - 1)\lambda_r$ new packets that have to be sent cross-channel. As before, denote by s_i the probability that an in-channel transmission is successful. Thus an in-channel packet is transmitted on the average $1/s_i$ times. Since such a packet is sent over L hops, it is transmitted on the average L/s_i times. The first of these hops is transmitted by the originating node. The remaining $L-1$ hops can be either a regular node or a bridge. Since the proportion of hops in which a regular node forwards the packet is $\dfrac{\lambda_r}{\lambda_r + \lambda_b}$, for each packet generated by a regular node and destined to a node in the same channel, the number of transmissions by a regular node is given by

$$I_1 = \frac{(\lambda_r + \lambda_b)}{(C\lambda_r + \lambda_b)}\left[\frac{1}{s_i} + \frac{L-1}{s_i}\frac{\lambda_r}{\lambda_r + \lambda_b}\right] = \frac{1}{s_i}\,\frac{L\lambda_r + \lambda_b}{(C\lambda_r + \lambda_b)} \qquad (22)$$

Similarly, the number of packets transmitted by a regular node to a bridge in order to be forwarded on another channel is

$$I_2 = \frac{1}{s_c} \frac{(C - 1)\lambda_r}{(C \lambda_r + \lambda_b)} \tag{23}$$

where s_c is the success probability for a cross-channel transmission (sent to a bridge selected at random). Similar arguments show that the number of packet transmissions by regular nodes of a packet generated by a bridge is

$$I_3 = \frac{(L-1)}{s_i} \frac{\lambda_r}{\lambda_r + \lambda_b} \frac{(\lambda_r + \frac{\lambda_b}{C})}{(C \lambda_r + \lambda_b)} = \frac{(L-1)}{s_i} \frac{1}{C} \frac{\lambda_r}{\lambda_r + \lambda_b} \tag{24}$$

and the number of packets generated in a different channel and transmitted by a regular node is

$$I_4 = \frac{(L-1)}{s_i} \frac{(C-1)\lambda_r}{C\lambda_r + \lambda_b} \frac{\lambda_r}{\lambda_r + \lambda_b} \tag{25}$$

From the above equations, it follows that

$$P_{in} = \frac{\lambda_r (I_1 + I_4) + \lambda_b I_3}{\lambda_r (I_1 + I_2 + I_4) + \lambda_b I_3} . \tag{26}$$

Using the above expression for P_{in} we can now derive the equations for the throughput of a regular node.

$$s_i(r,\theta) = e^{-(\lambda_r p_r + \lambda_b p_b) A(r,\theta)} , \tag{27}$$

$$f(r,\theta) = \frac{\lambda r e^{-\lambda R (\cos^{-1} t - t\sqrt{1-t^2})}}{1 - e^{-\lambda \pi R^2/2}} , \tag{28}$$

where $\lambda = \lambda_r + \lambda_b$, and $t = \frac{r \cos\theta}{R}$.

$$s_i = \int_0^R \int_{-\pi/2}^{\pi/2} s_i(r,\theta) f(r,\theta) \, d\theta \, dr , \tag{29}$$

$$S_{ir} = p_r P_{in} \left[1 - e^{-\lambda \pi R^2/2}\right] \left[1 - \frac{\lambda_r p_r + \lambda_b p_b}{\lambda}\right] s_i , \tag{30}$$

$$S_{cr} = p_r \ (1 - P_{in}) (1 - p_b) (1 - e^{-\lambda_b \pi R^2}) \ e^{-(\lambda_r \ p_r \ + \lambda_b \ p_b) \pi R^2} \ , \tag{31}$$

$$S_{regular} = S_{ir} + S_{cr} \ . \tag{32}$$

To calculate the throughput of a bridge at a specific channel we first observe that all transmissions by a bridge are in-channel and are routed by MFR since the bridge has the routing information for all channels. Thus, for a bridge $P_{in} = 1$. The probability of successful transmission by a bridge is the same as s_i for a regular node. This is the throughput of a bridge on each of the C channels for which it has a transceiver.

The total throughput for all channels per unit area is

$$S = C \ \lambda_r \ S_r + C \lambda_b \ S_b \ . \tag{33}$$

This should be compared to the throughput per unit area for a receiver-directed transmission scheme where the density of nodes is $C \ \lambda_r + \lambda_b$.

The calculation of expected progress is made in the same way as for the receiver-directed architecture. Transmissions from a regular node to a bridge in order to switch channels make no progress on average, since they are routed at random. In-channel transmissions make progress whether they are from a regular node or a bridge. The expected progress is given by

$$Z = \left[\frac{p_r \ P_{in} \ \lambda_r \ + p_b \ \lambda_b}{\lambda} \right] \left[1 - e^{-\lambda \pi R^2/2} \right] \tag{34}$$

$$\cdot \left[1 - \frac{\lambda_r \ p_r \ + \lambda_b \ p_b}{\lambda} \right] \int_0^R \int_{-\pi/2}^{\pi/2} r \cos\theta s_i (r, \theta) \ f \ (r, \theta) \ d\theta \ dr \ .$$

IV. NUMERICAL RESULTS

It is clear from the discussion above that there are many possible combinations of architecture, routing, and channel-access protocol to be considered, many of them can be evaluated from the models presented in this paper. We have selected a set of graphs that demonstrates the effects of the various choices on the network performance. For clarity we present and discuss these effects only in terms of throughput. The forward progress can be similarly calculated from the models given in the paper.

These results are given in Figures 5-10 which depict the throughput as a function of the total node density. Different curves represent different choices of protocol and number of channels. For each density, the value of the throughput is the maximum for all possible p 's -- the probability of a node transmitting in a slot. Transmission radius $R = 1$ is used for all figures. The figures can be interpreted in two ways, either:(1) given a particular density of nodes, transmission power, and number of channels, they show the throughput that can be attained, or:(2) given a set of nodes and number of channels, one can choose the transmission power so that the optimum number of radios is within range. Most of the figures show the throughput per *node*, per slot. This is the throughput perceived by an individual node in the network. The global performance of the network, on the other hand, can be measured using the throughput per *unit area* per slot.

A. Slotted ALOHA with MFR Routing

Figure 5 shows the throughput per node per slot for a multichannel slotted ALOHA system, using MFR routing in-channel and selecting the neighbor for cross-channel transmission at random. Systems with 1, 2, 3, 4, and 5 channels are shown. At high densities of nodes, the throughput per node grows almost linearly with the number of channels available. The optimum

value of throughput is almost identical for any number of channels. However, the density at which the optimum is achieved corresponds to approximately the same density of nodes in each quiescent channel.

The throughput per channel per unit area is displayed in Figure 6 for systems with 1, 2, 3, 4, and 5 channels. The system throughput is clearly limited by the throughput per channel. The throughput increases almost linearly at low densities, corresponding to the increase in the probability of a receiver being found in the forward direction. As the density of nodes increases, the throughput asymptotically approaches a limit, $1/e$, equal to the throughput in a single-hop, fully connected network.

Observe that optimum throughput per node is achieved at densities somewhat lower than those for which maximum throughput per unit area is achieved. At the density where the throughput per node achieves its maximum, the throughput per unit area is relatively small compared to its possible maximum, which implies that optimizing the individual node's performance does not necessarily lead to globally optimal performance.

In order to evaluate the contribution of routing information to the throughput, Figure 7 depicts the throughput per node under both partial-information and full-information routing schemes. Each of these schemes is represented by curves for 1, 3, and 5 channels. Recall that, under full-information, all traffic is routed under MFR whereas, under partial-information routing, only in-channel traffic is sent under MFR while cross-channel traffic is forwarded either to a random neighbor or to the nearest neighbor. Figure 7 shows that a large increase in the maximum throughput is possible when full routing is used compared to the throughput achieved under partial information. In high densities, however, both schemes perform about equally with little advantage to the partial routing because of the smaller interference when packets are sent to the nearest neighbor. Notice also, that the extra overhead needed to obtain the full routing information was not included in the calculation, hence the net advantage of this scheme is somewhat smaller than is predicted by Figure 7. Also notice that the transmitting cross-channel traffic to

the nearest neighbor is always better than transmitting to a neighbor at random.

Figure 8 depicts the performance of the PRNET under slotted ALOHA with perfect capture and without capture. All curves are for MFR routing for in-channel traffic and random selection of neighbor for cross-channel packets. In low-to-moderate densities, capture increases the throughput noticeably; however, at high densities the effect is much weaker.

1. Effect of Adding Transceivers (Bridges)

The performance of the architecture with bridges is shown in Figure 9, which contains 3 sets of curves, one for a network where 0.9 of the nodes are bridges, another where this ratio is 0.1, and a third consisting of a single-transceiver per node, receiver-directed architecture. The major conclusions from this figure are: (1) adding hardware to the network to make bridges pays off in terms of higher throughput, (2) the gain in throughput is significant in low-to-moderate densities, and (3) at high densities all the schemes perform virtually the same.

B. CSMA with MFR Routing

Figure 10 shows the throughput per node under the CSMA protocol both for perfect capture and for no capture at all. Whereas, in the previous figures, more channels represent adding bandwidth to the network, here the total bandwidth is assumed to be constant and split into subchannels. As mentioned above, for a fixed packet length, the ratio of propagation delay to the packet transmission time decreases as the bandwidth decreases, which implies that the CSMA performance will improve as the channel is partitioned into a larger number of subchannels. This effect is demonstrated in Figure 10 where the throughput is normalized to the number of channels to account for the constant total bandwidth available. Notice that, at medium-to-large densities, the throughput per slot per node increases with the number of channels. The effect is much more marked when capture is available. At low densities, a smaller number of channels seems to be preferable because there the event of not finding a neighbor to transmit to has high probability, which increases as the number of channels the node population is partitioned into increases.

V. CONCLUSIONS

We have proposed architectures for a multihop packet radio network that has several channels available. The performance has been calculated under a variety of protocols. The throughput per node can be maintained, as the density of nodes increases, by adding extra channels. The total network system throughput is limited by the number of channels available and achieves its maximum value at higher node densities than the maximum per node throughput.

When the propagation delay is long relative to the packet length, a significant performance gain can be achieved by splitting the channel into multiple channels and using a CSMA protocol. The effect of FM capture is to accentuate the improvements obtained when no capture is present. Several other architectures and routing schemes are being studied and will form the subject of a future paper.

VI. APPENDIX: AREAS with CAPTURE

We calculate the area within the circle of interference that is known to have no radios because of the "most forward" routing. There are four cases to consider, depending on the position (r, θ) of a receiver relative to a transmitter, and on the value of α, the capture parameter. [We use $r' = \min[\alpha r, R]$ and $\theta' = \cos^{-1}\frac{r\cos\theta}{R}$.] When there is no capture, $r' = R$, so that the no-capture cases are easily deduced from the following formulas.

The area that contains potentially interfering radios is a circle of radius r' , except that, because of most forward routing, a portion may be known to be empty. $X(r,\theta)$ is the area containing no radios. $Y(r,\theta)$ is the area that includes radios within the circle of interference at the receiver, but outside the transmission radius of the transmitter; these radios may therefore interfere with a CSMA transmission.

We first calculate $Y(r,\theta)$. This area is determined with reference to Figure 11. It is the area outside the transmission radius of P, but within the circle of interference; it is also the area within which radios, although unable to hear P's transmission, will nevertheless be powerful enough to interfere at Q.

$$\omega = \cos^{-1}\frac{R^2+r^2-r'^2}{2rR}$$

$$\phi = \cos^{-1}\frac{r'^2+r^2-R^2}{2rr'}$$

$$\text{area } \Delta \text{ PQI} = \frac{1}{2}rR\sin\omega$$

$$\text{area segment PIN} = \frac{1}{2}\omega R^2$$

$$\text{area segment QIN'} = \frac{1}{2}(\pi-\phi)r'^2$$

$$Y(r,\theta) = 2\cdot(\text{ QIN'} + \Delta\text{PQI} - \text{PIN })$$

$$= (\pi-\phi)r'^2 + rR\sin\omega - \omega R^2$$

The area $X(r,\theta)$ can now be found, with reference to Figure 12.

(a) $r' + r < R$:

The receiver is so close that the whole circle of interference lies within the radius of transmission of the transmitter

$$X(r,\theta) = \frac{\pi}{2} r'^{\,2} \tag{35}$$

The other cases require more calculation.

(b) $r \sin\theta + r' < R \sin\theta'$:

The circle of interference extends beyond the radius of transmission, but does not include either end of the chord through the receiver; the chord is normal to the direction of progress. The area we require is the same as case a), reduced by the area outside the transmission radius, $Y(r,\theta)$.

$$X(r,\theta) = (\phi - \frac{\pi}{2}) r'^{\,2} + \omega R^2 - rR \sin\omega \tag{36}$$

(c) $r' < r \sin\theta + R \sin\theta'$: The circle of interference includes one end of the perpendicular chord, but not both ends.

$$\text{area of sector PIK} = \frac{1}{2} R^2 (\omega - \theta')$$

$$\text{area of sector QIJ} = \frac{1}{2} r'^{\,2} (\frac{\pi}{2} - \phi + \theta)$$

$$\text{area of } \Delta \text{ PKQ} = \frac{1}{2} rR \sin(\theta' - \theta)$$

$$\text{area of } \Delta \text{ PIQ} = \frac{1}{2} rR \sin\omega$$

$$\text{area of IJK} = \frac{1}{2} \left[r'^{\,2} (\frac{\pi}{2} - \phi + \theta) + rR (\sin\omega - \sin(\theta' - \theta)) + R^2 (\theta' - \omega) \right]$$

Hence,

$$X(r,\theta) = \frac{1}{2}\Big(r'^{\,2}(\theta+\phi-\frac{\pi}{2}) + R^2(\omega+\theta' -\theta) \tag{37}$$

$$- rR\,(\sin(\theta' -\theta) + \sin\omega)\Big)$$

(d) The circle of interference includes both ends of the perpendicular chord.

$$X(r,\theta) = R^2\theta' - rR\sin\theta' \cos\theta' \tag{38}$$

A. Areas for Nearest Neighbor Cross Channel

Consider a transmission to the nearest neighbor, which is located at (r,θ), with transmission power such that any simultaneous transmission within distance R of the receiver will interfere. If there is capture, then the same formulas apply with R replaced by r'. There are two cases to consider.

(a) $2r < R$

In this case, the circle known to contain no radios lies completely within the circle of potentially interfering radios around the receiver.

$$X(r,\theta) = \pi r^2$$

(b) In this case, the circle known to be without transmitters is not completely within the circle of interference. It is seen by referring to Figure 11, that the area known to be without transmitters, but outside the circle of interference is the same as the area of hidden nodes in the case of CSMA with perfect capture. Hence

$$X(r,\theta) = \phi r^2 + \omega R^2 - rR\sin\omega$$

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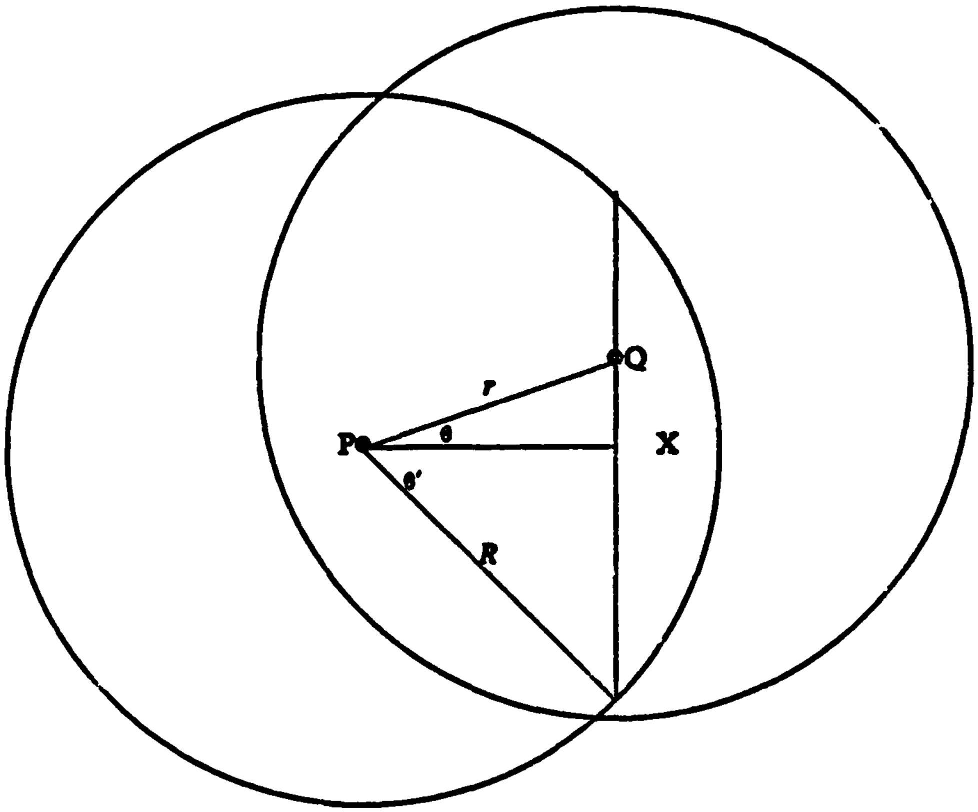

Figure 1.
MFR Routing

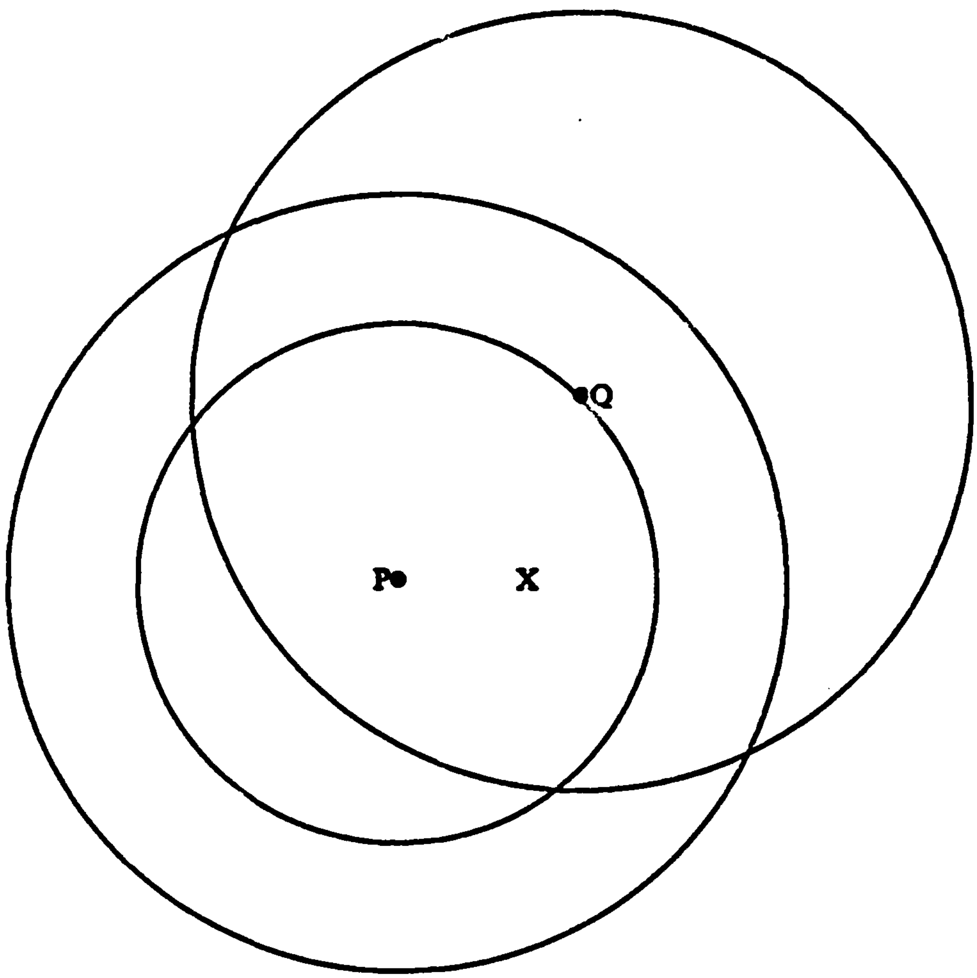

Figure 2.
Nearest Neighbor Routing

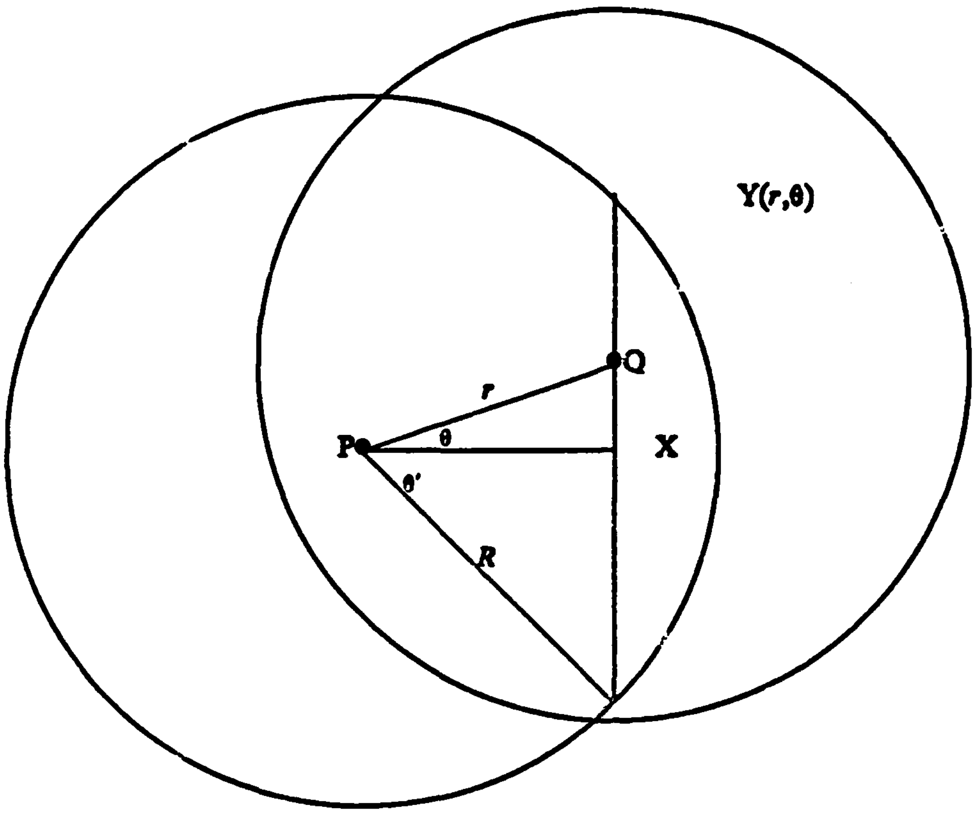

Figure 3.
CSMA MFR Routing

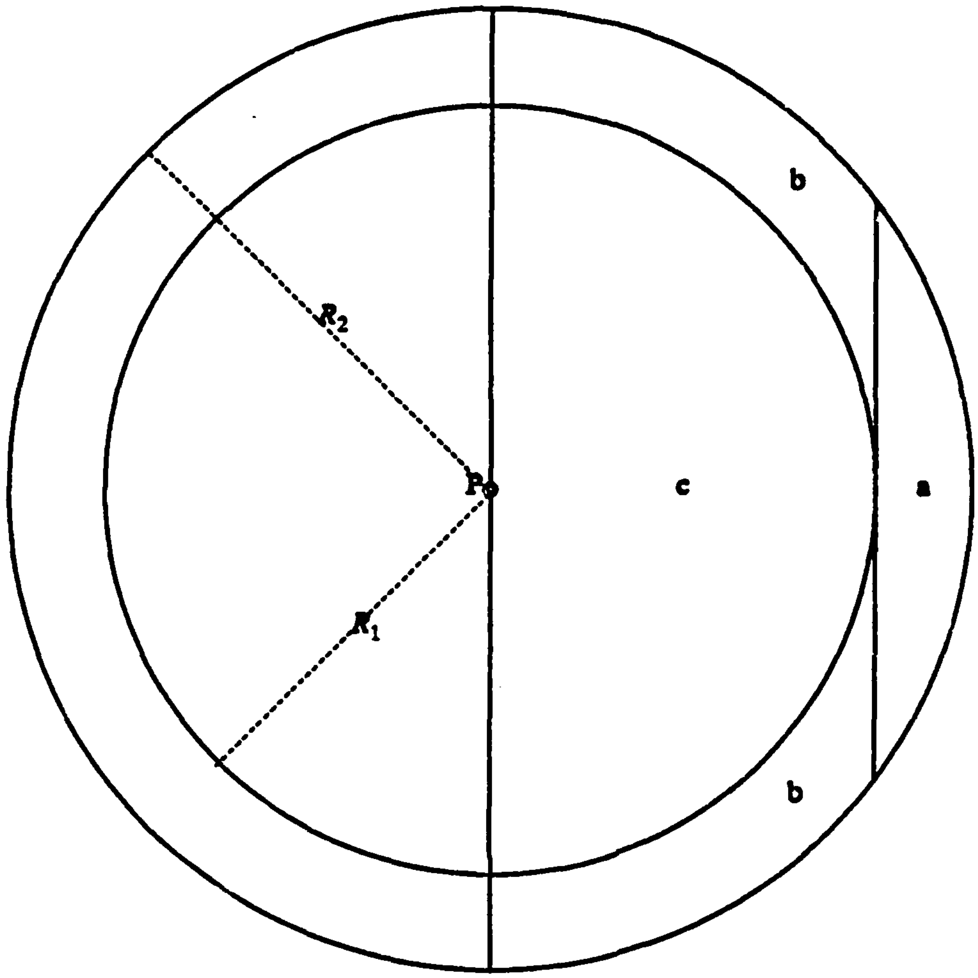

Figure 4.
Full Information Routing
Different Transmission Powers

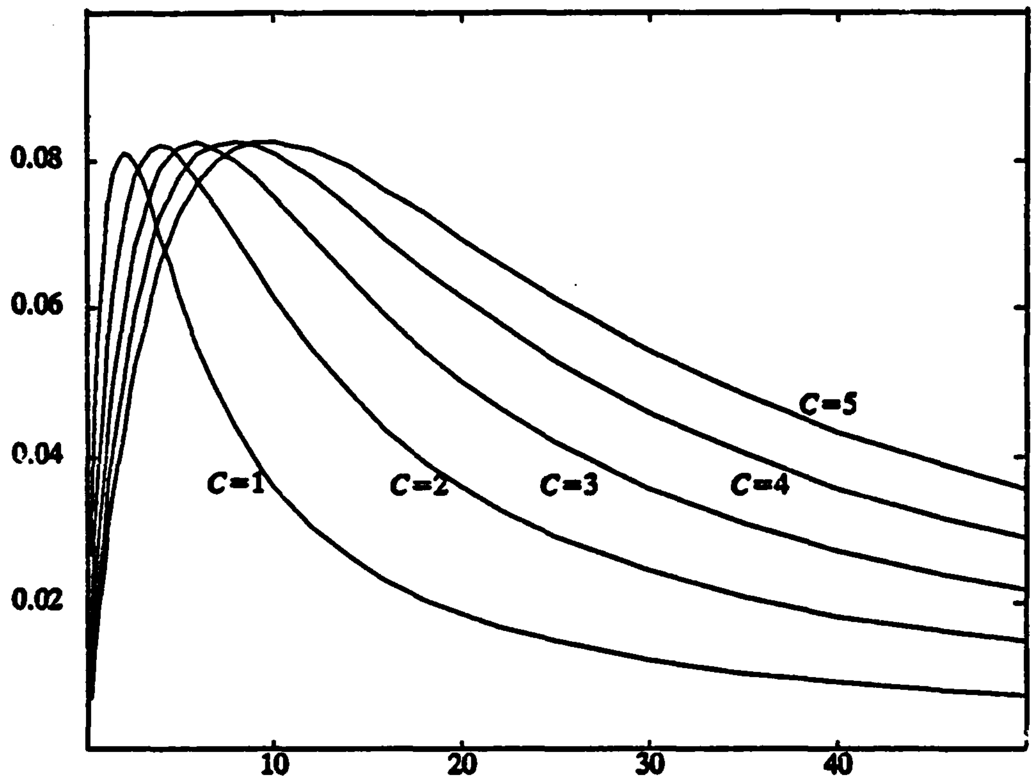

Figure 5.
Throughput Per Radio of Multichannel Slotted ALOHA
Partial Information Routing (Random Cross-Channel)

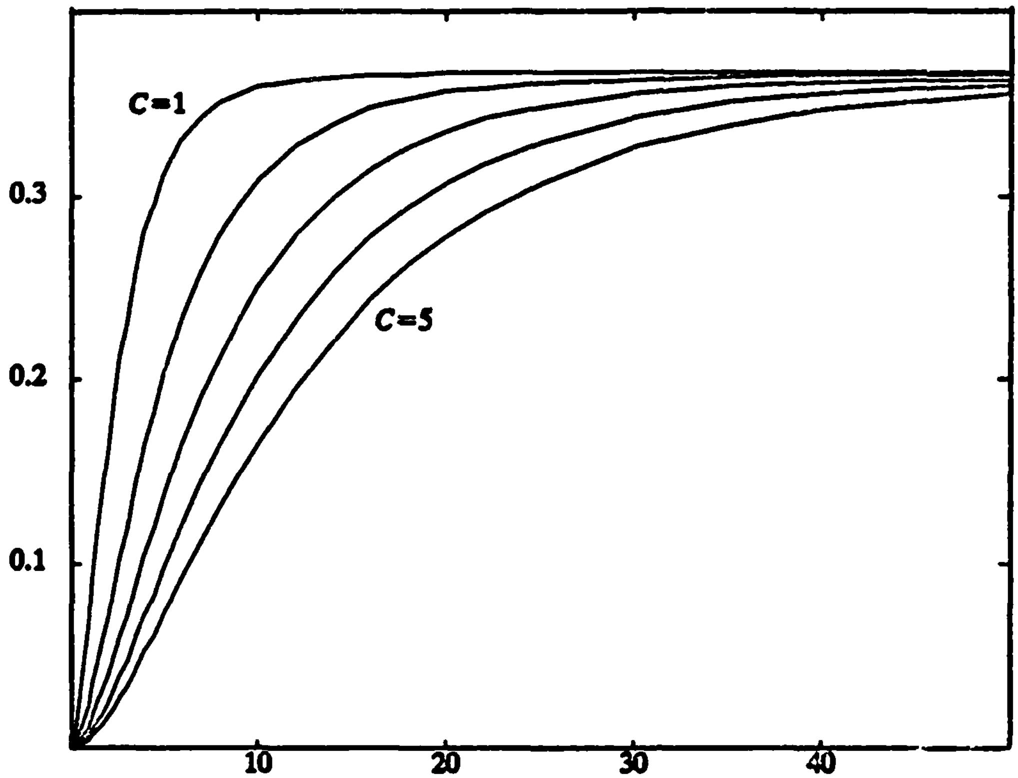

Figure 6.
Throughput Per Channel Per Unit Area
Slotted Aloha MFR
Partial Information Routing (Random Cross-Channel)

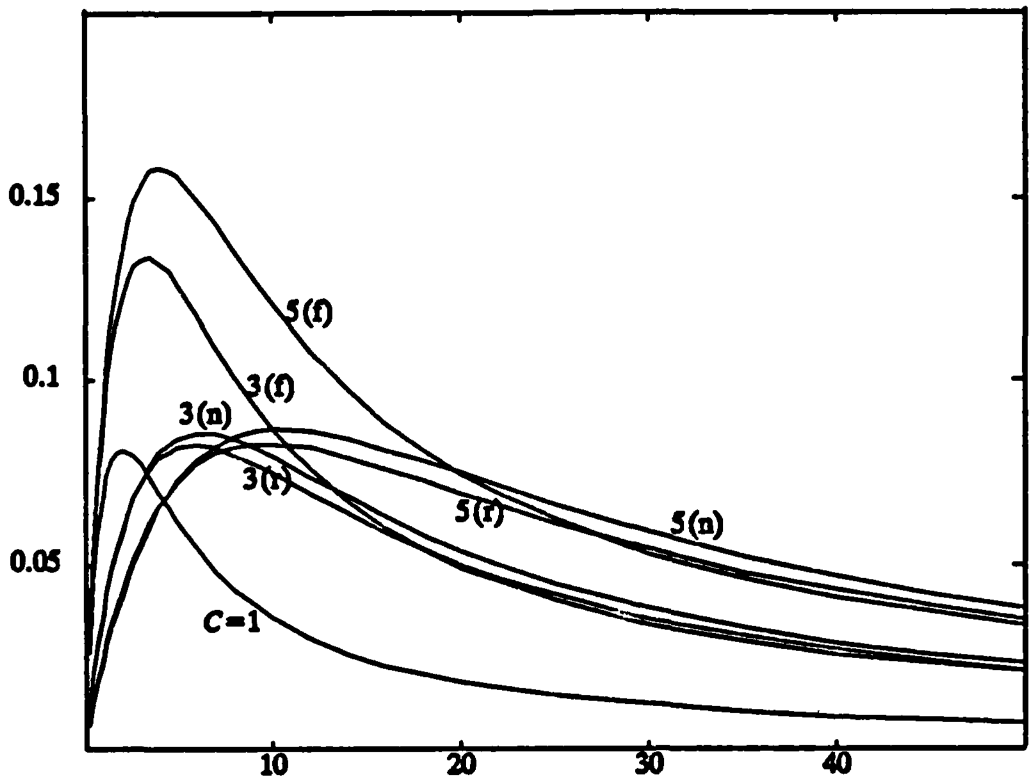

Figure 7.
Throughput of Slotted ALOHA with Different Routing Strategies
Full Information Routing
Partial Information Routing (Nearest Cross Channel)
Partial Information Routing (Random Cross Channel)

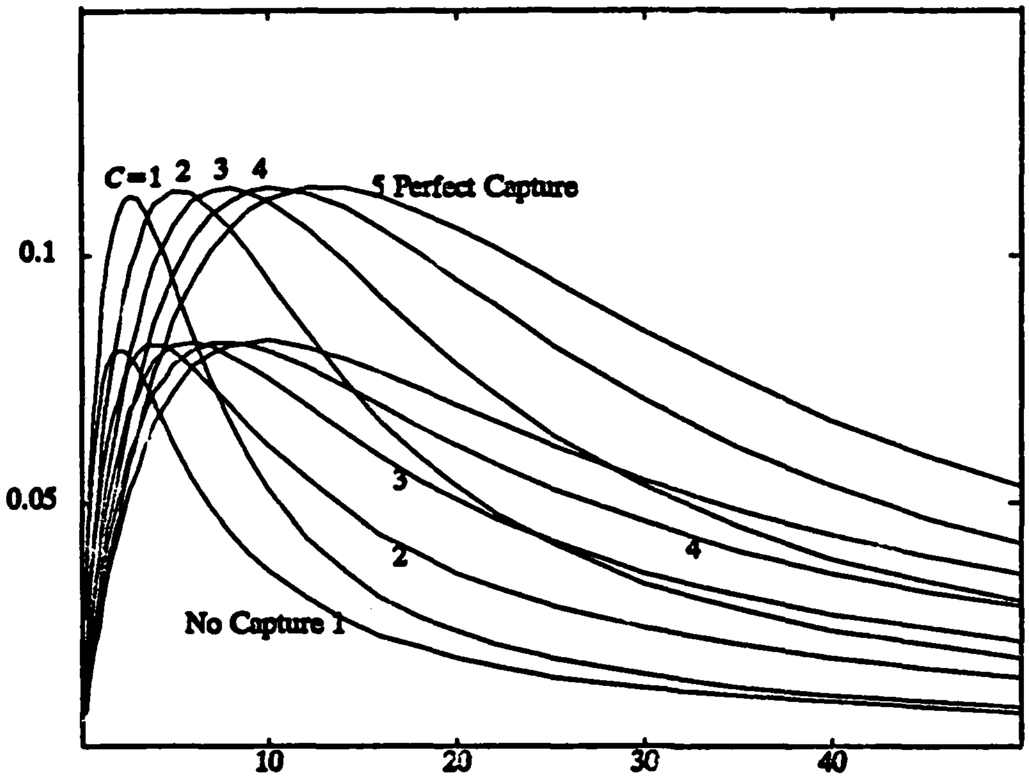

Figure 8.
Effect of Capture on Slotted ALOHA

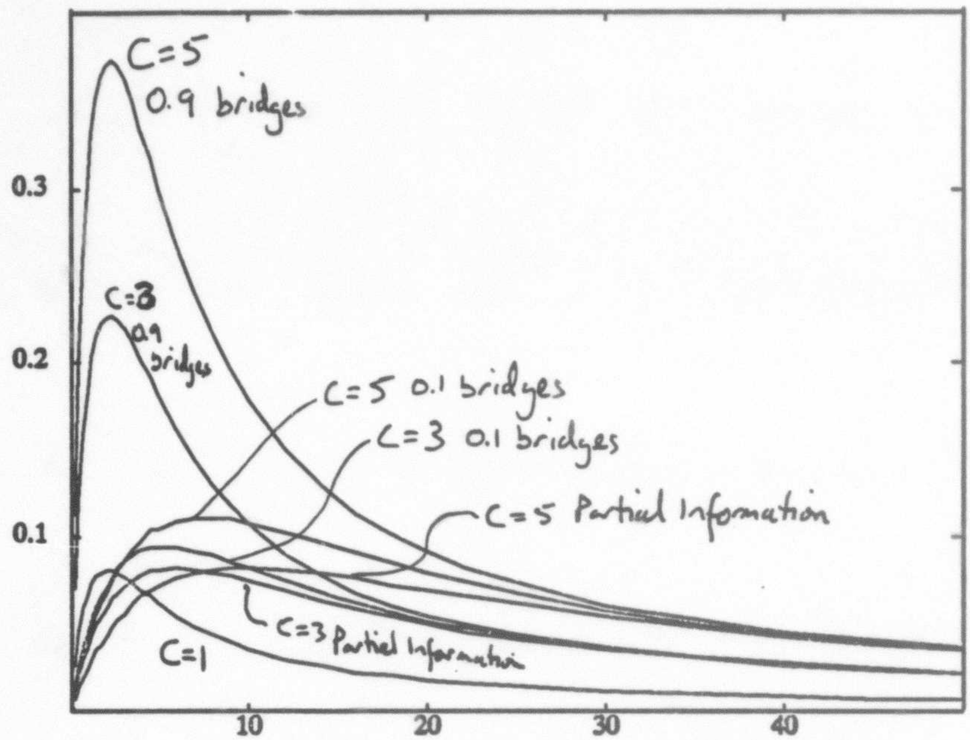

Figure 9.
Throughput of Slotted ALOHA
Partial Information
Bridges (0.1 of nodes are bridges)
Bridges (0.9 of nodes are bridges)

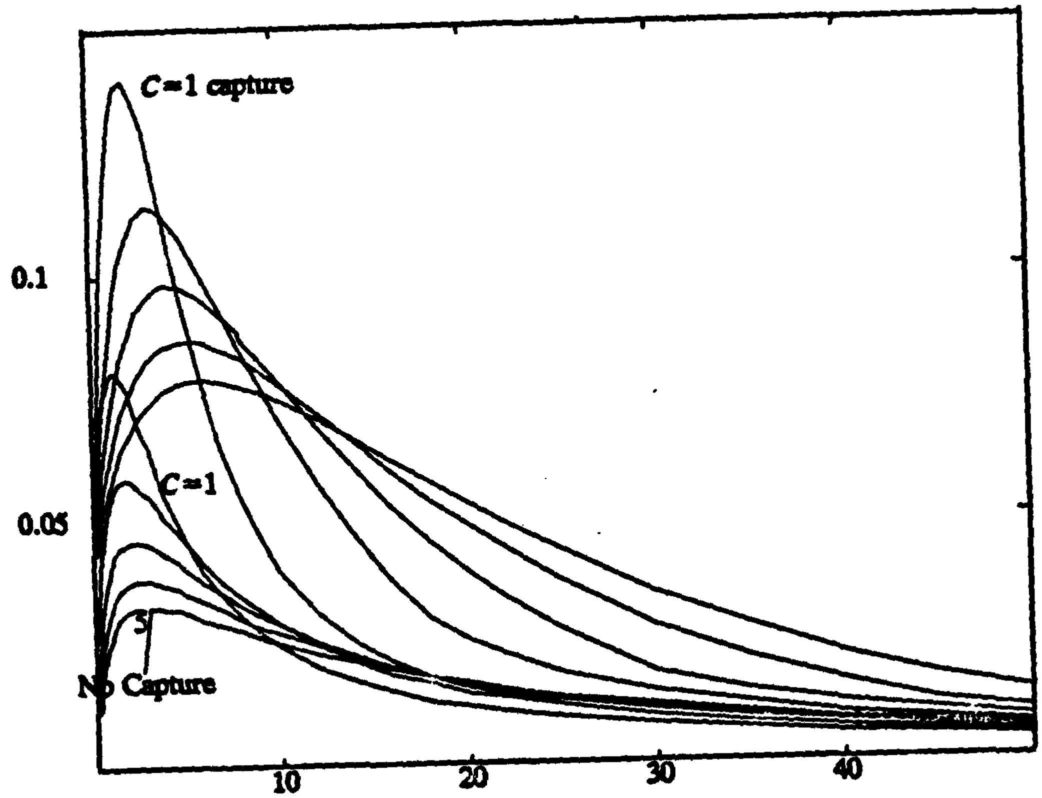

Figure 10.
Throughput of CSMA under Channel Splitting
No Capture and Perfect Capture
Propagation delay 0.5 of packet length (1 channel system)

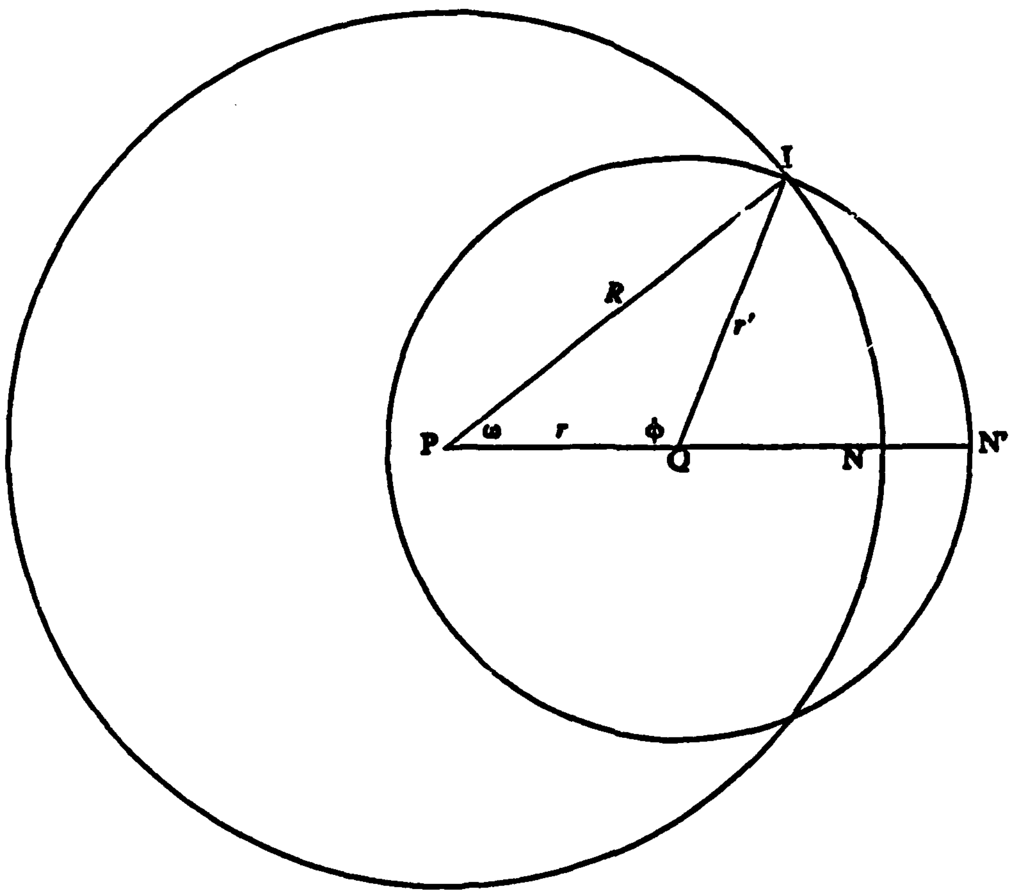

Figure 11.

CSMA Area of Interference

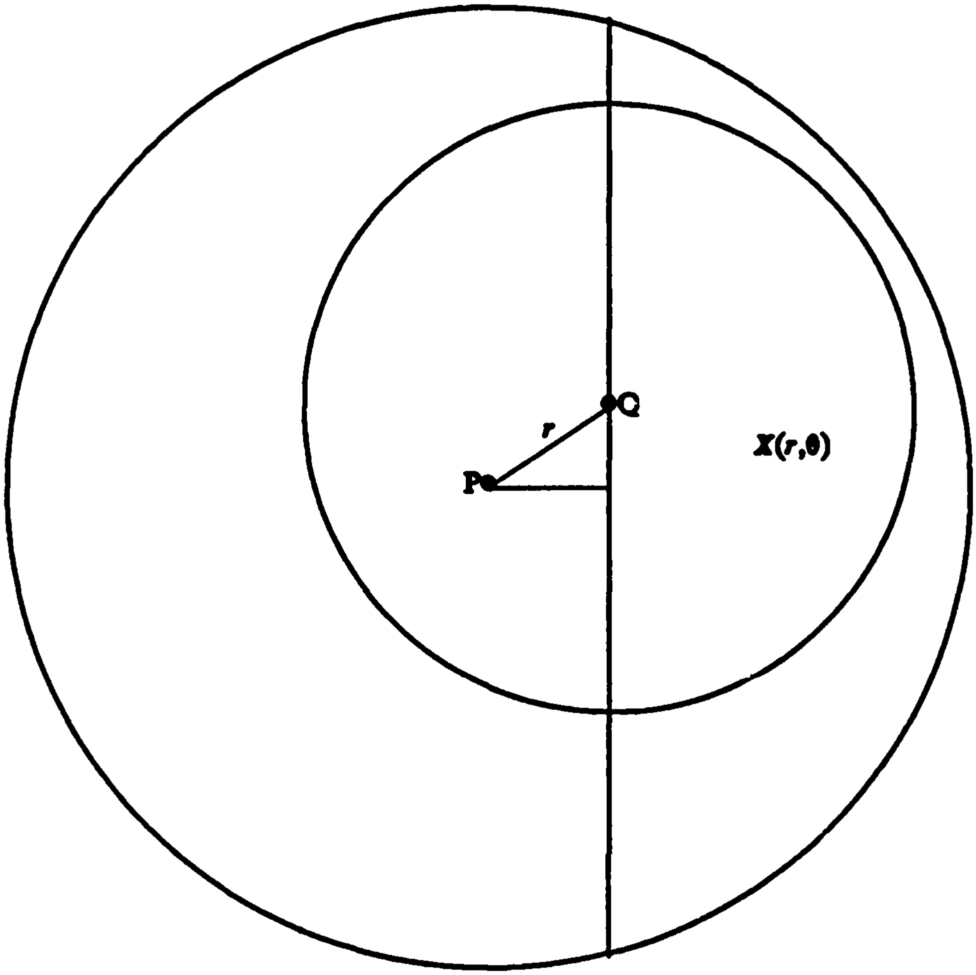

Figure 12(a).

$r'+r<R, \ \alpha=2$

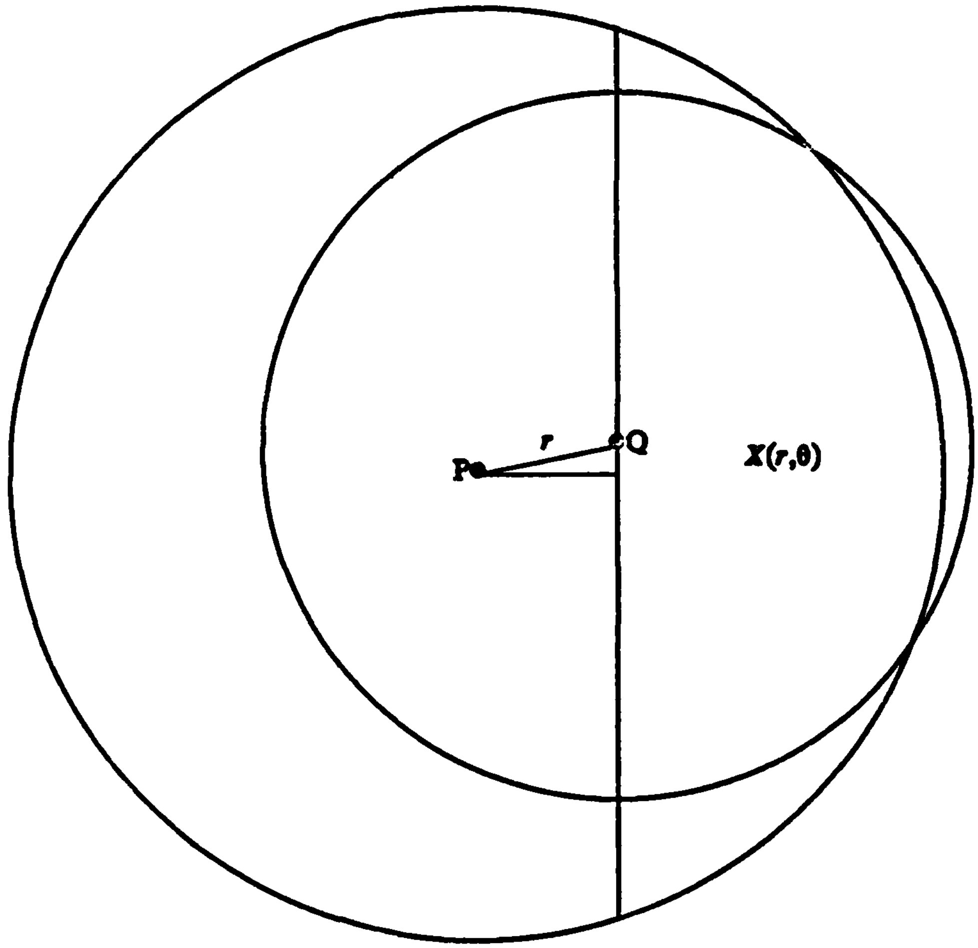

Figure 12(b).

$r' + r\sin\theta < R\sin\theta'$, $\alpha = 2$

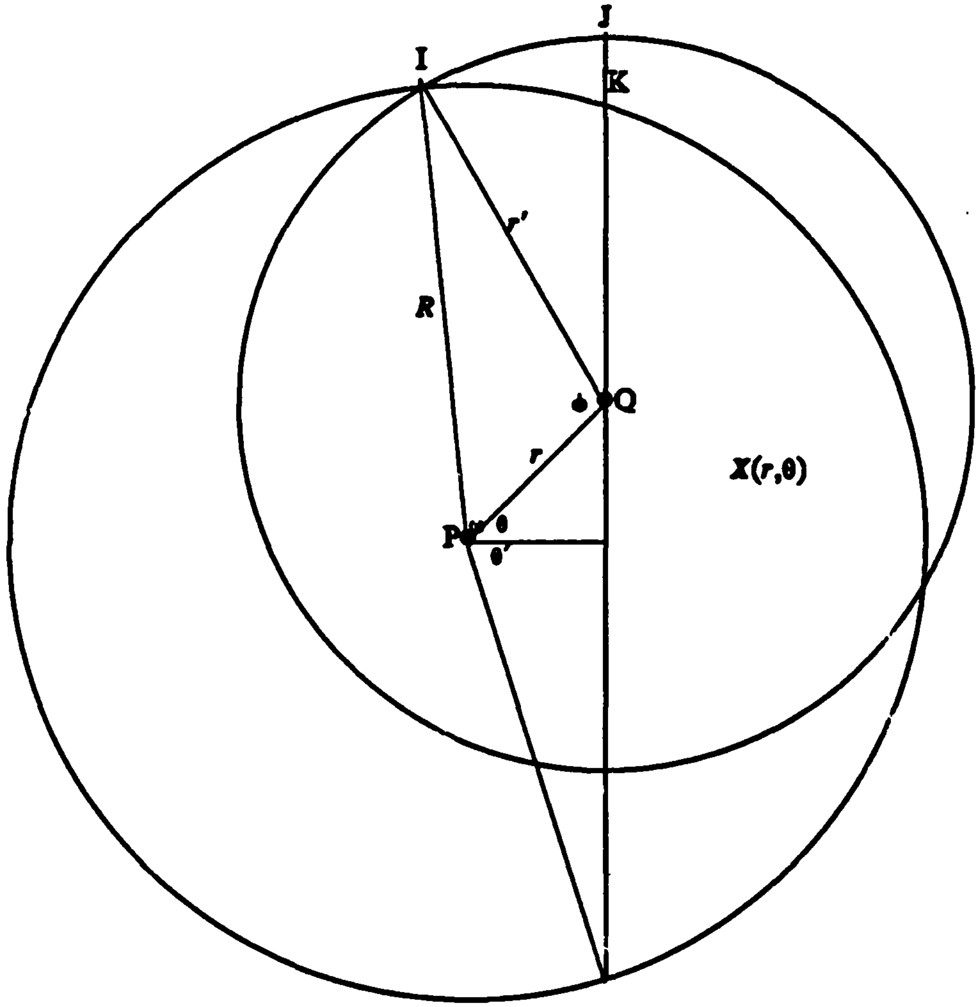

Figure 12(c).

$r' < r\sin\theta + R\sin\theta'$, $\alpha=2$

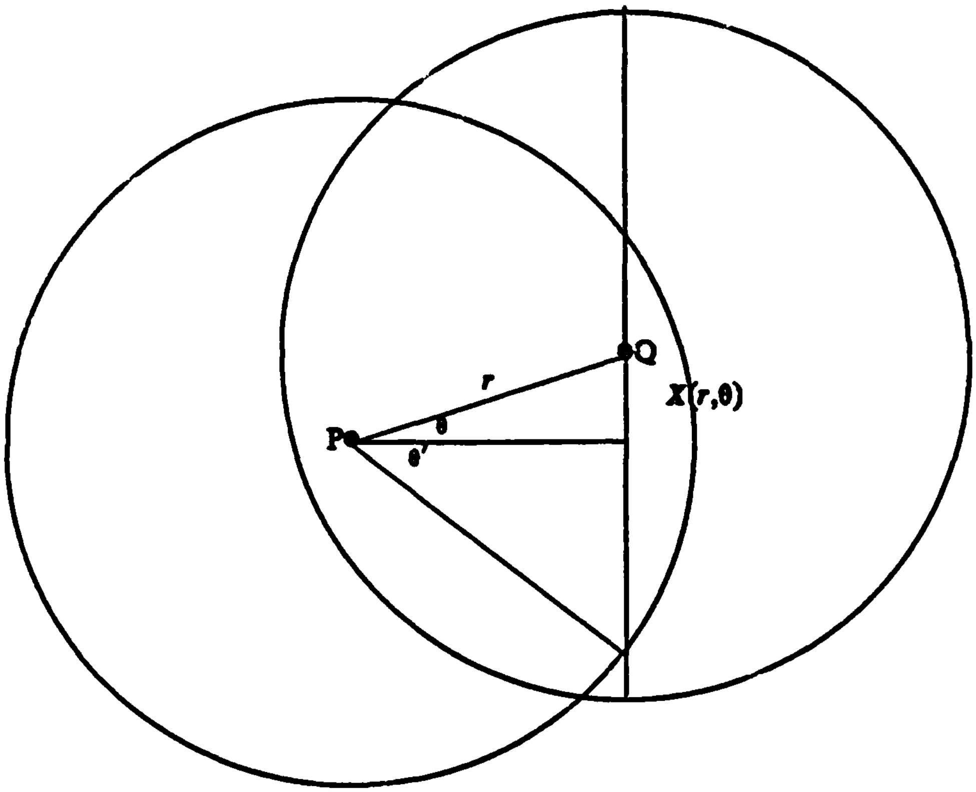

Figure 12(d).

$r' > r\sin\theta + R\sin\theta', \ \alpha = 2$

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